The Gain-Loss Spread:
A New and Intuitive Measure of Risk

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Abstract

The standard deviation, arguably the most widely-used measure of risk, suffers from at least two limitations. First, the number itself offers little insight; after all, what is the intuition behind the square root of the average quadratic deviation from the arithmetic mean return? Second, investors tend to associate risk more with bad outcomes than with volatility. To overcome these limitations, this article introduces a new measure of risk, the gain-loss spread (GLS), which is both intuitive and based on magnitudes that investors consider relevant when assessing risk. The evidence reported below shows that the GLS is highly correlated with the standard deviation, thus providing basically the same information about risk; and more correlated to mean returns than both the standard deviation and beta, thus providing a tighter link between risk and return.

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1. Introduction

The standard deviation is arguably the most widely-used measure of risk.[1] The problem is that, although very widely used, this magnitude is far from intuitive. After all, if the annual returns of an asset have a standard deviation of 20%, what does that number exactly mean? Technically, 20% is the square root of the average quadratic deviation from the arithmetic mean return. But that neither sounds nor is very intuitive.

Because this technically-correct interpretation of the standard deviation does not seem to be very insightful, most investors usually resort to thinking about this magnitude in relative terms. Thus, an asset with a volatility of 20% is riskier than one with a volatility of 15% and less risky than another with a volatility of 25%. However, although this approach may be useful when considering the *relative* risk of different assets, the actual magnitude used to assess the risk of each asset still suffers from lack of intuition.

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[1] According to financial theory, the appropriate measure of risk is the standard deviation when considering an asset in isolation, and beta when considering an asset that is part of a fully-diversified portfolio. Practitioners typically use these two magnitudes accordingly.

Importantly, this lack of intuition is far from harmless. Goldstein and Taleb (2007) find that finance practitioners seem to confuse the standard deviation with the mean absolute deviation, which leads them to underestimate risk. This underestimation of risk ranges between 25% in normally-distributed assets and up to 90% for assets with fat tails.

But the problems of the standard deviation do not end there. As is well known, when assets display skewness or kurtosis, the standard deviation is at best limited and at worst misleading as a measure of risk. Furthermore, most investors associate risk not necessarily with volatility but more narrowly with bad outcomes, such as losing money, or the probability of losing money, or the probability of falling short of a target return. In other words, investors tend to associate risk with the downside they face, which may help explain the increasing popularity of measures of downside risk; see, for example, Estrada (2002, 2004, 2006, 2007, 2008) and references therein. For this reason, the downside should be an *explicit* part of any magnitude that intends to properly reflect the way investors think about and assess risk.

The ultimate goal of this article is to propose a new measure of risk that is both intuitive and based on magnitudes that investors consider relevant when assessing risk. Such measure is the *gain-loss spread* (GLS) which, as discussed below, takes into account the probability of a loss, the average loss, and the average gain.

Importantly, as the evidence reported below shows, the GLS is highly correlated with the standard deviation of returns, thus providing basically the same information, as well as more insight, than the most widely-used measure of risk. Furthermore, the evidence reported below shows, first, that the GLS is more correlated with mean returns than both the standard deviation and beta, thus providing a tighter link between risk and return; and, second, that it is able to discriminate between high-return and low-return portfolios better than beta and just as well or better than the standard deviation, thus being a useful tool for portfolio selection.

The rest of this article is organized as follows. Section 2 provides a very brief review of measures of risk. Section 3 introduces the GLS both formally and through examples. Section 4 reports and discusses evidence supporting the plausibility of the GLS as a measure of risk. Finally, section 5 concludes with an assessment. An appendix at the end contains tables that describe the data and some of their characteristics.

2. A (Very) Brief Review of Measures of Risk

This section provides a very brief overview of risk measures. It does not aim to be exhaustive and it does not focus on the evolution of the concept of risk; rather, it highlights some magnitudes that have been proposed to assess it. Both Bernstein (1996) and Holton (2004) provide insightful historical perspectives on the evolution of the concept of risk in finance.

As is well known, the first formal definition of risk, the standard deviation of returns, was proposed by Markowitz (1952, 1959). This magnitude is a measure of total risk and aims to capture dispersion around the mean return. The higher the dispersion, the higher the uncertainty, and therefore the higher the risk of the asset considered.

Part of this total risk can be eliminated through diversification by pooling assets into a portfolio. The risk that cannot be diversified away is the systematic risk and is measured by beta. This magnitude, proposed by Sharpe (1964), Lintner (1965), and Mossin (1966), with credit also given to Treynor (1961), who never published his work, aims to capture volatility relative to the market and measures whether an asset magnifies or mitigates the market's fluctuations.

Markowitz (1959) also pioneered the use of downside risk measures by supporting the semideviation, which measures volatility below a chosen benchmark. Other measures of downside risk include the lower partial moment of Bawa (1975) and Fishburn (1977), which basically generalizes the semideviation;[2] value at risk or VaR, pioneered at J. P. Morgan, which measures the worst expected outcome over a chosen time horizon at a chosen level of confidence; and downside beta, proposed by Estrada (2002, 2007), which measures whether an asset magnifies or mitigates the market's downside fluctuations.[3] Nawrocki (1999) provides a brief history of downside risk measures and Estrada (2006) provides a primer on the topic aimed at practitioners.

Biglova et al (2004) review other risk measures relevant for optimal portfolio selection, including the mean absolute deviation, the stable dispersion measure, Gini's mean difference, conditional value at risk, and the mini-max. Researchers in behavioral finance, however, argue that risk is a concept too complicated to be summarized by a single magnitude, and therefore propose to use not just one but several factors instead; see, for example, Fisher and Statman (1999).

3. The Issue

This section starts by introducing the gain-loss spread with an example in order to highlight the intuition behind this new measure of risk. It then introduces the GLS formally, and

[2] The semideviation with respect to a chosen benchmark B (Σ_B) is given by $\Sigma_B = \{(1/T)\cdot\Sigma_t \text{ Min}(R_t - B)^2\}^{1/2}$, where R denotes returns, T the number of observations, and t indexes time. The lower partial moment with respect to B (LPM_B) is given by $LPM_B = (1/T)\cdot\Sigma_t \text{ Min}(R_t - B)^a$, where a is the degree of the lower partial moment. Thus, the lower partial moment does not restrict the deviations with respect to B to be quadratic and in that sense generalizes the semideviation.

[3] Hogan and Warren (1974), Bawa and Lindenberg (1977), and Harlow and Rao (1989) all propose CAPM-like models based on different definitions of downside beta.

then discusses another example to add further perspective on this magnitude. Finally, it generalizes the definition of gains and losses so that the GLS can accommodate any benchmark return.

3.1. An Example

In order to understand the underlying idea behind the measure of risk proposed in this article, a simple example may help. Exhibit 1 displays the annual returns of the MSCI World index, the most widely-used proxy for the world market (equity) portfolio, over the 20-year period between 1988 and 2007. The arithmetic mean and standard deviation of these returns are 10.7% and 15.7%, the latter figure being the square root of the average quadratic deviation with respect to the former figure. Arguably, this 15.7%, by itself, provides little insight about the risk of investing in the world market portfolio.

Exhibit 1: MSCI World, 1988-2007

This exhibit shows the annual returns of the MSCI World index between 1988 and 2007. Returns are in dollars and account for capital gains and dividends. All returns in %.

Year	Return	Year	Return	Year	Return	Year	Return
1988	24.0	1993	24.9	1998	22.0	2003	34.6
1989	17.6	1994	5.0	1999	26.8	2004	15.8
1990	−16.5	1995	19.5	2000	−13.9	2005	11.4
1991	19.9	1996	13.2	2001	−15.9	2006	21.5
1992	−4.2	1997	15.0	2002	−19.0	2007	12.2

A good starting point in the development of a more insightful measure of risk is to explicitly consider the variables that investors view as relevant when assessing the risk of an asset. Two of these variables focus on the downside. Investors are typically concerned about the probability of suffering losses, a magnitude that can be estimated with the proportion of periods in which an asset delivered negative returns. Assuming that the short history of returns in Exhibit 1 is representative of long-term behavior, the probability of an annual loss when investing in the world market portfolio is 25%. This number simply follows from the fact that this index delivered negative returns in 5 years (1990, 1992, 2000, 2001, and 2002) out of 20.

When assessing the downside of an asset, investors are not only concerned about the probability but also about the size of the potential losses, a magnitude that can be estimated with the mean return over the periods in which the asset delivered negative returns. Assuming again that the short history of returns in Exhibit 1 is representative of long-term behavior, then the average annual loss when investing in the world market portfolio is −13.9%. This number is simply the mean return over the five years in which the market went down; that is, (−16.5%−4.2%−13.9%−15.9%−19.0%)/5 = −13.9%.

The probability of a loss and the average loss straightforwardly lead to the expected loss, which for the world market portfolio is (25%)(−13.9%) = −3.5%. Note that this figure is not the expected annual *return* but the expected annual *loss*. Note, also, that this figure accounts for *both* the probability and the size of annual losses.

Similar calculations can be made when assessing the upside of an asset. Thus, the short history of returns in Exhibit 1 suggests that, when investing in the world market portfolio, the probability of an annual gain is 75% (the index delivered positive returns in 15 years out of 20), and the average annual gain is 18.9% (the mean return of the 15 years in which the market went up). Therefore, the expected gain is (75%)(18.9%) = 14.2%.

Finally, the gain-loss spread (GLS) of the world market portfolio is simply the difference between the expected gain and the expected loss; that is, GLS = 14.2%−(−3.5%) = 17.6%. This figure measures the spread or distance between an expected gain of 14.2%, a number that accounts for both the probability of a gain and the average gain, and an expected loss of −3.5%, a number that accounts for both the probability of a loss and the average loss.

Why is a GLS of 17.6% more insightful than a standard deviation of 15.7%? Simply because although there seems to be little intuition behind a 15.7% that reflects the square root of the average quadratic deviation with respect to the arithmetic mean return, the 17.6% reflects the spread between the upside and the downside, each of which in turn reflects what can be expected in terms of gains and losses from investing in the world market portfolio. In short, then, the GLS is an insightful magnitude built from the ground up by putting together three variables that investors consider relevant when assessing the risk of an asset, namely, the probability of a loss, the average loss, and the average gain.

3.2. Analytical Framework

The previous example shows that calculating a GLS is not only insightful but also very simple; this section formally introduces this measure of risk. As usual, the formalization below assumes that the magnitudes that make up the GLS are estimated from historical data. This of course does not prevent a subsequent adjustment of these magnitudes to reflect an investor's views about the future.

Consider an asset with returns R_t where the subscript t indexes time. Assume that, of the T periods for which returns are available, the asset delivers a loss $L_t = R_t < 0$ in N periods and a gain $G_t = R_t > 0$ in M periods, such that $N+M = T$. The probability of a loss (p_L) is then simply defined as

$$p_L = N/T \,, \tag{1}$$

and the probability of a gain as $p_G = M/T = 1-p_L$.[4]

The average loss (A_L) is defined as the mean return over the N periods in which the asset delivered a negative return; that is,

$$A_L = (1/N)\cdot\sum_{t=1}^{N} L_t \ . \tag{2}$$

Similarly, the average gain (A_G) is defined as the mean return over the M periods in which the asset delivered a positive return; that is,

$$A_G = (1/M)\cdot\sum_{t=1}^{M} G_t \ . \tag{3}$$

The expected loss (E_L) follows from (1) and (2) and is defined as

$$E_L = p_L \cdot A_L = (1/T)\cdot\sum_{t=1}^{N} L_t \ . \tag{4}$$

Similarly, the expected gain (E_G) is defined as

$$E_G = p_G \cdot A_G = (1/T)\cdot\sum_{t=1}^{M} G_t \ . \tag{5}$$

Finally, the gain-loss spread (GLS), the magnitude introduced in this article, is defined as the difference between the expected gain and the expected loss; that is,

$$GLS = E_G - E_L = (1/T)\cdot\left\{\sum_{t=1}^{M} G_t - \sum_{t=1}^{N} L_t\right\} . \tag{6}$$

As this expression clearly shows, calculating a GLS is very simple.[5] And as the example in the previous section hopefully shows, it provides more insight about the risk of an asset than the widely-used standard deviation.

3.3. Another Example

In order to add further perspective on the gain-loss spread, Exhibit 2 shows the GLS and related magnitudes for the S&P-500 over the 1900-2007 period. As the magnitudes based on annual returns show, the S&P delivered losses in 27.8% of the years (30 years), with an average annual loss of −13.5%, resulting in an expected annual loss of −3.7%. The expected annual gain of 15.4%, on the other hand, results from an average annual gain of 21.3% in the 72.2% of the

[4] It is implicitly assumed that the asset delivers either gains or losses; hence, there are no periods in which R_t =0. This is plausible for most assets even at the daily frequency. Still, it is trivial to accommodate in this framework S periods with R_t =0, such that $N+M+S = T$.

[5] Note that although E_G-E_L yields the GLS, E_G+E_L yields the arithmetic mean return.

years (78 years) in which the S&P delivered gains. The annual GLS of 19.1% reflects the spread between the annual expected gain of 15.4% and the annual expected loss of −3.7%.

Exhibit 2: S&P-500, 1900-2007

This exhibit shows information about the annual and monthly return series of the S&P-500 index over the 1900-2007 period. The first four magnitudes show the number of observations (*T*), the arithmetic (*AM*) and geometric (*GM*) mean return, and the standard deviation (*SD*). The next three show the probability of a loss (p_L), the average loss (A_L), and the expected loss (E_L). The next three show the probability of a gain (p_G), the average gain (A_G), and the expected gain (E_G). The last magnitude is the gain-loss spread (GLS). All magnitudes as defined in section 3.2. Returns are in dollars and account for capital gains and dividends. All figures but *T* in %.

	T	*AM*	*GM*	*SD*	p_L	A_L	E_L	p_G	A_G	E_G	GLS
Annual	108	11.6	9.8	19.6	27.8	−13.5	−3.7	72.2	21.3	15.4	19.1
Monthly	1296	0.9	0.8	5.1	38.3	−3.6	−1.4	61.5	3.7	2.3	3.7

The monthly GLS of 3.7%, in turn, results from the spread between an expected monthly gain of 2.3% and an expected monthly loss of −1.4%. The latter results from the 38.3% of the months in which the S&P delivered losses and an average monthly loss of −3.6%; the former, in turn, results from the 61.5% of the months in which the S&P delivered gains and an average monthly gain of 3.7%.[6]

Finally, the comparison of the annual GLS of the S&P (19.1%) and that of the world market portfolio estimated in section 2.1 (17.6%) leads to the conclusion that the US market is riskier than the world market. This is unsurprising given that the latter is more diversified, but it is reassuring that the GLS points in the expected direction.

3.4. Generalizing Gains and Loses

The discussion in the previous sections defines gains and losses as positive and negative returns, which amounts to set 0 as the benchmark with respect to which gains and losses are measured. However, some investors may be interested in benchmarks other than 0, such as the rate of inflation, the risk-free rate, or a target return, among many other possibilities.

The framework discussed can be straightforwardly generalized to define gains and losses with respect to any arbitrary benchmark *B*. In this case, gains and losses can be redefined as $G_t = R_t - B > 0$ and $L_t = R_t - B < 0$, respectively. These definitions may be useful to investors that, for example, have a target return they want to meet on a periodic basis.

This more general definition of gains and losses can be further generalized to account for a time-varying benchmark B_t. In this case, gains and losses can be redefined as $G_t = R_t - B_t > 0$ and $L_t = R_t - B_t < 0$, respectively. These definitions may be useful to investors that, for example, focus on real returns (in which case B_t would be the periodic rate of inflation) or on returns above the periodic risk-free rate.

[6] The S&P delivered neither gains nor losses (R_t =0%) in 2 of the 1296 months in the sample.

4. The Evidence

This section discusses the evidence supporting the gain-loss spread as a measure of risk. It first tests the ability of the GLS to explain the cross-section of returns in countries and industries. Then it evaluates the in-sample ability of the GLS to discriminate between high-return and low-return portfolios. And finally considers two investable strategies to evaluate the out-of-sample ability of the GLS as a tool for portfolio selection.

4.1. Statistical Significance: The Cross-Section of Returns

In order to test the ability of the GLS to explain the cross-section of returns, the entire MSCI database of countries and industries was used. The database contains monthly data on 49 countries (22 developed and 27 emerging) and 57 industries. The full sample period available for every country and industry was used in the estimations. Although not all series start at the same time, in all cases the data goes through Dec/07. Exhibit A1 in the appendix lists all the countries and industries in the sample, the month in which return data begins for each of the them, and some summary statistics. Exhibit A2, also in the appendix, reports the GLS and related magnitudes of all the countries and industries in the sample.

The first step of the analysis consisted of estimating, over the whole sample period available for each variable, the (arithmetic) mean return, standard deviation, beta with respect to the MSCI World index, and GLS of every country and industry in the sample. Subsequently, cross-section regressions were run with mean return as the dependent variable and combinations of the three risk variables (standard deviation, beta, and GLS) as independent variables. These regressions were run across countries, across industries, and across a pooled sample of countries and industries. The results for the cross-section of countries are summarized in Exhibit 3.

EXHIBIT 3: Cross-Section Analysis – Countries

This exhibit shows cross-section regressions between mean returns (the dependent variable) and combinations of three risk variables, the standard deviation (SD), beta with respect to the MSCI World index (Beta), and the gain-loss spread (GLS). The countries in the cross section are those shown in Exhibit A1 in the appendix. All parameters calculated over the whole sample period available for each country. Returns are monthly, in dollars, and account for capital gains and dividends. Significance is based on White's heteroskedasticity-consistent covariance matrix. The critical value for a one-sided test at the 5% significance level is 1.68.

	Constant	*t*-stat	SD	*t*-stat	Beta	*t*-stat	GLS	*t*-stat	R^2
Panel A	0.003	2.265	0.136	7.746					0.556
	0.008	2.906			0.008	2.607			0.178
	0.002	1.102					0.205	7.962	0.578
Panel B	0.001	0.795	−0.043	−0.426			0.269	1.791	0.579
	0.001	0.452			0.002	0.776	0.194	7.898	0.583
Panel C	0.001	0.297	−0.039	−0.393	0.002	0.761	0.252	1.755	0.584

Panel A shows the results of simple regressions in which mean returns are regressed on each of the three risk variables, one at a time. As this panel shows, all three variables have the expected sign and are statistically significant. Of the three, the GLS is the one that has the highest explanatory power as measured by the R^2. Panel B shows that when the GLS and the standard deviation are jointly considered, only the GLS is significant and has the expected sign; and when the GLS and beta are jointly considered, again only the GLS is significant and has the expected sign. Finally, panel C shows that when all three risk variables are jointly considered, only the GLS is significant and has the expected sign. Therefore, Exhibit 3 suggests that when explaining mean returns across countries, the GLS outperforms both the standard deviation and beta.

Exhibit 4 shows the results of a similar analysis for the cross-section of industries. When each risk variable is considered individually (panel A), only the GLS and the standard deviation are significant and have the expected sign; of these two, the GLS has a higher explanatory power as measured by the R^2. When the GLS and the standard deviation are jointly considered (panel B), only the GLS is significant and has the expected sign; similar results are obtained when the GLS and beta are considered together. Finally, when all three risk variables are jointly considered (panel C), only the GLS is significant and has the expected sign (the standard deviation and beta have the wrong sign and are significant). Therefore, Exhibit 4 suggests that when explaining mean returns across industries, the GLS again outperforms both standard deviation and beta.

EXHIBIT 4: Cross-Section Analysis – Industries

This exhibit shows cross-section regressions between mean returns (the dependent variable) and combinations of three risk variables, the standard deviation (SD), beta with respect to the MSCI World index (Beta), and the gain-loss spread (GLS). The industries in the cross section are those shown in Exhibit A1 in the appendix. All parameters calculated over the whole sample period available for each industry. Returns are monthly, in dollars, and account for capital gains and dividends. Significance is based on White's heteroskedasticity-consistent covariance matrix. The critical value for a one-sided test at the 5% significance level is 1.67.

	Constant	*t*-stat	SD	*t*-stat	Beta	*t*-stat	GLS	*t*-stat	R^2
Panel A	0.005	4.468	0.065	2.901					0.133
	0.008	4.546			0.001	0.460			0.009
	0.004	3.565					0.108	3.708	0.169
Panel B	0.001	0.307	−0.535	−2.455			0.889	3.001	0.300
	0.004	4.140			−0.006	−3.772	0.231	5.710	0.306
Panel C	0.001	0.804	−0.462	−2.381	−0.005	−3.850	0.888	3.407	0.401

Finally, Exhibit 5 shows the results of a similar analysis for a pooled sample of countries and industries, and again with similar results. When considered individually (panel A), the GLS is significant, has the expected sign, and achieves the highest explanatory power as measured by the R^2. When the GLS is jointly considered with either the standard deviation or beta (panel B), only the GLS is significant and has the expected sign. And when all three variables are jointly considered (panel C), again only the GLS is significant and has the expected sign. Therefore, Exhibit 5 suggests that when explaining mean returns across both countries and industries, the GLS outperforms both standard deviation and beta.

EXHIBIT 5: Cross-Section Analysis – Countries and Industries

This exhibit shows cross-section regressions between mean returns (the dependent variable) and combinations of three risk variables, the standard deviation (SD), beta with respect to the MSCI World index (Beta), and the gain-loss spread (GLS). The countries and industries in the cross section are those shown in Exhibit A1 in the appendix. All parameters calculated over the whole sample period available for each country and industry. Returns are monthly, in dollars, and account for capital gains and dividends. Significance is based on White's heteroskedasticity-consistent covariance matrix. The critical value for a one-sided test at the 5% significance level is 1.66.

	Constant	*t*-stat	SD	*t*-stat	Beta	*t*-stat	GLS	*t*-stat	R^2
Panel A	0.002	2.508	0.132	8.937					0.510
	0.009	4.887			0.003	1.648			0.044
	0.001	1.041					0.199	9.465	0.539
Panel B	−0.000	−0.043	−0.157	−1.350			0.428	2.578	0.550
	0.003	2.101			−0.003	−2.536	0.227	9.447	0.570
Panel C	0.002	1.076	−0.152	−1.373	−0.003	−2.547	0.448	2.758	0.581

The results reported in Exhibits 3-5 all point in the same and unequivocal direction: When explaining mean returns across countries, across industries, and across countries and industries combined, the GLS outperforms both the standard deviation and beta. Furthermore, because the GLS is almost perfectly correlated with the standard deviation, it provides basically the same information about risk.[7] In short, then, the GLS provides a more insightful measure of risk and a tighter link between risk and return than do both the standard deviation and beta.

4.2. Economic Significance: Return Spreads

The evidence just discussed clearly supports the GLS as a measure of risk, and for this reason an interesting question to ask is whether the GLS is not only statistically significant but also *economically* significant. In other words, does a portfolio of high-GLS assets outperform a portfolio of low-GLS assets by a substantial margin? Is this margin larger than that between high-risk and low-risk portfolios when risk is measured by standard deviation and beta? These are the questions addressed in this section.

As before, the first step of the analysis consisted of estimating, over the whole sample period available for each variable, the (arithmetic) mean return, standard deviation, beta with respect to the MSCI World index, and GLS of every country and industry in the sample. Subsequently, all countries were ranked by their standard deviation, and three equally-weighted portfolios were formed, the top third with the most volatile countries (P1) and the bottom third

[7] The correlation between the GLS and the standard deviation is 0.99 across countries, across industries, and across countries and industries combined. The correlation between the GLS and beta, in turn, is 0.47 across countries, 0.78 across industries, and 0.50 across countries and industries combined.

with the least volatile countries (P3). Finally, the spread in mean monthly returns (*S*) between these two portfolios was calculated and subsequently annualized (*AS*). The same process was repeated for countries ranked by beta and GLS; and then repeated again for industries, and for a pooled sample of countries and industries. All results are summarized in Exhibit 6.

Exhibit 6: Return Spreads

This exhibit shows mean returns and spreads in mean returns. Countries, industries, and countries and industries combined were ranked by their standard deviation (SD), beta with respect to the MSCI World index (Beta), and gain-loss spread (GLS). In each case, three equally-weighted portfolios were formed, the top third with the riskiest assets (P1) and the bottom third with the least risky assets (P3), and their mean returns were calculated. *S* denotes the spread between the mean return of P1 and P3, and *AS* denotes the annualized spread. The countries and industries in the sample are those shown in Exhibit A1 in the appendix. All parameters calculated from monthly returns and over the whole sample period available for each country and industry. Returns are in dollars and account for capital gains and dividends. All returns in %.

	Countries				Industries				Countries & Industries			
	P1	P3	*S*	*AS*	P1	P3	*S*	*AS*	P1	P3	*S*	*AS*
SD	1.9	1.1	0.8	9.8	1.0	0.9	0.1	1.7	1.7	0.9	0.7	8.9
Beta	1.8	1.5	0.4	4.3	0.9	1.0	−0.1	−1.0	1.4	1.2	0.1	1.4
GLS	2.0	1.1	0.9	11.4	1.0	0.9	0.2	2.0	1.7	0.9	0.8	9.4

The results for countries show that the spread in mean monthly returns between the most volatile and the least volatile portfolios is 0.8%, or 9.8% annualized. Such spread is higher than that observed between high-beta and low-beta portfolios (4.3% annualized) but lower than that observed between high-GLS and low-GLS portfolios (11.4% annualized). This last figure shows that differences in GLS may span very large differences in mean returns, and suggests that this risk measure has the potential to be profitably used as the basis of an investing strategy. (More on this below.) Furthermore, the 1.6% (=11.4%−9.8%) differential spread spanned by the GLS relative to that spanned by the standard deviation is far from negligible.

The results for industries, though less spectacular, confirm the ability of the GLS to discriminate between high-return and low-return assets. The annualized spread between high-GLS and low-GLS portfolios is 2.0%, higher than those spanned by the standard deviation (1.7%) and beta (perhaps surprisingly, −1.0%). Finally, the pooled sample of countries and industries confirms and strengthens the previous results. The annualized spread between high-GLS and low-GLS portfolios is a substantial 9.4%, higher than those spanned by the standard deviation (8.9%) and beta (1.4%).

In short, then, these results show that the GLS is able to discriminate between high-return and low-return portfolios better than both the standard deviation and beta. Furthermore, the spreads spanned by the GLS are substantial from an economic point of view, ranging from a low of 2% a year for industries to a high of 11.4% a year for countries. These results suggest that the GLS may be a useful tool for portfolio selection, an issue that is addressed immediately below.

4.3. Investable Strategies

The in-sample analysis of the previous section, although illuminating, does not evaluate the out-of-sample ability of the GLS, relative to that of standard deviation and beta, to discriminate between high-return and low-return portfolios. In order to complement the previous analysis, this section evaluates the usefulness of the GLS as a tool for portfolio selection by testing two out-of-sample (investable) strategies.

The first step of the analysis consisted of estimating risk parameters (SD, beta, and GLS) for every country and industry in the sample between the beginning of data coverage and Dec/99. Countries were then ranked by their estimated standard deviations and allocated into three equally-weighted portfolios, the top third consisting of the riskiest countries and the bottom third consisting of the least risky ones. Subsequently, two strategies were implemented, the first consisting of a long-only investment of $100 in the portfolio of riskiest countries, and the second consisting of a long-short investment, short $100 the portfolio of least risky countries and long $100 the portfolio of riskiest countries. Both the long-only and the long-short portfolios were passively held through the end of 2007 at which point they were liquidated.

The same process was then repeated for countries after being ranked by beta and by GLS; and then it was repeated again for industries after being ranked by standard deviation, beta, and GLS. The performance of all these portfolios is summarized in Exhibit 7.

Focusing on the long-only strategies first, the exhibit shows that risky portfolios based on the standard deviation and GLS performed similarly and better than those based on beta. In the case of countries, the performance of all three portfolios was similar in terms of terminal wealth, mean return, risk, and risk-adjusted return; in the case of industries, portfolios based on beta performed substantially worse in terms of all these magnitudes than portfolios based on standard deviation and GLS, both of which performed almost identically.

As for the long-short strategies, the most remarkable result is that low-beta portfolios outperformed high-beta portfolios in the case of both countries and industries. On the contrary, high-risk portfolios outperformed low-risk portfolios, as expected, when risk was measured by standard deviation and GLS. Portfolios based on these two measures of risk performed again very similarly in terms of terminal wealth, mean return, risk, and risk-adjusted return in the case of both countries and industries.

Exhibit 7: Investable Strategies

This exhibit shows the results of two investable strategies. For each country and industry in the sample the standard deviation (SD), beta with respect to the MSCI World index (Beta), and gain-loss spread (GLS) were estimated between the beginning of data coverage and Dec/99. Countries were then ranked by their estimated SDs and allocated into three equally-weighted portfolios; subsequently, $100 were invested in the countries with high SDs (the top third of the ranking), in one case as a long-only position and in the other as a long-short position with the proceeds coming from shorting the countries with low SDs (the bottom third of the ranking); the long-only and long-short portfolios were held through Dec/07, at which point their terminal value (TV), arithmetic mean return (AM), geometric mean return (GM), risk measures (SD, Beta, and GLS), and risk-adjusted return (RAR) were calculated. The same process was repeated after ranking countries by beta and by GLS; and then repeated again after ranking industries by SD, beta, and GLS. RAR is defined as the ratio between AM and the indicated risk measure. 'Long-Only' denotes the portfolios with high SD, beta, and GLS, and 'Long-Short' denotes portfolios short low-risk assets and long high-risk assets. The countries and industries in the sample are those shown in Exhibit A1 in the appendix. All magnitudes but TV are monthly figures. All figures in % except for TVs (in $) and betas.

	Long-Only			Long-Short		
	SD	Beta	GLS	SD	Beta	GLS
Countries						
TV	359.70	348.01	353.38	159.32	73.26	157.36
AM	1.51	1.46	1.50	0.56	−0.28	0.56
GM	1.34	1.31	1.32	0.49	−0.32	0.47
SD	5.74	5.44	5.88	3.92	3.07	4.07
Beta	1.14	1.13	1.17	0.21	0.35	0.27
GLS	4.97	4.72	5.07	3.17	2.40	3.28
RAR-SD	0.26	0.27	0.25	0.14	−0.09	0.14
RAR-Beta	0.01	0.01	0.01	0.03	−0.01	0.02
RAR-GLS	0.30	0.31	0.30	0.18	−0.11	0.17
Industries						
TV	205.52	158.75	204.37	116.38	66.31	111.36
AM	0.84	0.60	0.83	0.18	−0.38	0.14
GM	0.75	0.48	0.75	0.16	−0.43	0.11
SD	4.13	4.72	4.17	2.27	3.12	2.37
Beta	0.99	1.17	1.00	0.24	0.56	0.19
GLS	3.41	3.87	3.44	1.59	2.37	1.67
RAR-SD	0.20	0.13	0.20	0.08	−0.12	0.06
RAR-Beta	0.01	0.01	0.01	0.01	−0.01	0.01
RAR-GLS	0.25	0.15	0.24	0.12	−0.16	0.08

These results show that the GLS can be profitably used as the basis of an investable strategy, and that its out-of-sample ability to discriminate between high-risk and low-risk portfolios is better than that of beta and similar to that of the standard deviation. They therefore strengthen the results of the previous section and confirm the plausibility of the GLS as a proper measure of risk.

5. Assessment

Risk can be defined in many ways; some even claim that it simply is in the eyes of the beholder. Still, of the many magnitudes devised to assess it, the standard deviation of returns is the most widely used. The problem is, however, that it lacks a clear intuition. The number itself provides little insight into the risk of an asset, and most academics and practitioners typically use it more in relative terms (the larger the standard deviation, the riskier the asset) than in absolute terms (interpreting a specific value of this magnitude).

Compounding this problem is the fact that investors tend to associate risk less with volatility and more with downside factors, such as losing money, or the probability of losing money, or the probability of falling short of a target return. For this reason, the downside should be an explicit part of an intuitive measure of risk.

The magnitude proposed in this article, the gain-loss spread, is built from the ground up by putting together variables that investors do associate with risk. By combining the probability of suffering losses, the magnitude of the potential loses, and the magnitude of the potential gains, the GLS provides a more intuitive measure of risk. Furthermore, because it is highly correlated with the standard deviation, it basically provides the same information about risk; and it does so providing, at the same time, more insight about the risk of an asset.

The evidence reported shows that the GLS is more correlated with mean returns than both the standard deviation and beta; this is the case across countries, across industries, and across countries and industries combined. In other words, the GLS is not only more insightful than the standard deviation but also provides a tighter link between risk and return. Furthermore, the evidence shows that the GLS is able to discriminate between high-return and low-return portfolios better than beta and just as well or better than the standard deviation, thus being a useful tool for portfolio selection.

In short, then, this article proposes a new measure of risk that is more intuitive than other competing and widely-used magnitudes. Admittedly, it is an ad-hoc measure. But it does provide intuitive insight into the risk of an asset; it is based on magnitudes that investors do consider relevant when assessing risk; and it is supported by the evidence. Goldstein and Taleb (2007) express their hope that "one day, Finance will adopt a more natural metric than standard deviation." Perhaps the GLS proposed in this article is the tool they were hoping for.

Appendix

Exhibit A1: Summary Statistics

This exhibit shows, for the series of monthly returns, the arithmetic mean (AM), standard deviation (SD), and beta with respect to the MSCI World index (Beta) of all the countries and industries in the sample, all calculated between the beginning (Start) and the end (Dec/2007) of each variable's sample period. All country and industry benchmarks are MSCI indices. Returns are in dollars and account for capital gains and dividends. AM and SD in %.

Country	AM	SD	Beta	Start	Industry	AM	SD	Beta	Start
Developed					Aerospace & defense	1.3	5.4	0.9	Jan/95
Australia	1.1	6.8	1.0	Jan/70	Air freight & logistics	0.9	4.9	0.8	Jan/95
Austria	1.1	5.9	0.5	Jan/70	Airlines	0.6	6.0	1.1	Jan/95
Belgium	1.3	5.4	0.8	Jan/70	Auto components	0.7	4.9	0.9	Jan/95
Canada	1.1	5.5	1.0	Jan/70	Automobiles	0.8	5.5	1.1	Jan/95
Denmark	1.3	5.3	0.7	Jan/70	Beverages	0.9	4.2	0.6	Jan/95
Finland	1.6	9.1	1.4	Jan/88	Biotechnology	0.9	8.5	0.8	Jan/95
France	1.2	6.3	1.0	Jan/70	Building products	0.5	4.9	0.9	Jan/95
Germany	1.1	6.0	0.9	Jan/70	Chemicals	1.0	4.5	0.9	Jan/95
Hong Kong	1.9	10.5	1.2	Jan/70	Commercial banks	0.9	4.9	1.0	Jan/95
Ireland	1.0	5.6	1.0	Jan/88	Commercial services & supplies	0.5	4.3	1.0	Jan/95
Italy	0.9	7.1	0.8	Jan/70	Communications equipment	1.2	9.7	2.0	Jan/95
Japan	1.1	6.3	1.0	Jan/70	Computers & peripherals	1.3	8.0	1.6	Jan/95
Netherlands	1.3	5.2	1.0	Jan/70	Construction & engineering	0.8	5.3	0.8	Jan/95
New Zealand	0.8	6.5	0.8	Jan/88	Construction materials	0.9	5.0	1.0	Jan/95
Norway	1.4	7.5	1.0	Jan/70	Containers & packaging	0.3	5.4	0.9	Jan/95
Portugal	0.8	6.3	0.8	Jan/88	Distributors	0.0	8.3	1.3	Jan/95
Singapore	1.4	8.3	1.1	Jan/70	Diversified financial services	1.0	5.4	1.2	Jan/95
Spain	1.1	6.3	0.9	Jan/70	Diversified telecommunication services	0.7	5.5	1.1	Jan/95
Sweden	1.4	6.7	1.0	Jan/70	Electric utilities	1.0	3.2	0.4	Jan/95
Switzerland	1.1	5.2	0.9	Jan/70	Electronic equipment & instruments	0.5	7.6	1.5	Jan/95
UK	1.1	6.4	1.1	Jan/70	Electronic equipment manufacturers	1.0	5.5	1.2	Jan/95
USA	0.9	4.3	0.9	Jan/70	Energy equipment & services	1.6	8.5	1.2	Jan/95
Emerging					Food products	0.9	3.4	0.4	Jan/95
Argentina	2.7	16.2	0.7	Jan/88	Food/staples retailing	0.7	3.4	0.5	Jan/95
Brazil	3.1	15.6	1.6	Jan/88	Gas utilities	1.0	3.8	0.6	Jan/95
Chile	1.8	7.0	0.7	Jan/88	Health care equipment & support	1.0	4.0	0.6	Jan/95
China	0.7	10.9	1.1	Jan/93	Health care providers & services	0.9	5.5	0.5	Jan/95
Colombia	1.8	9.3	0.5	Jan/93	Hotels, restaurants & leisure	0.8	4.6	0.9	Jan/95
Czech Rep.	1.9	8.0	0.7	Jan/95	Household durables	0.5	6.1	1.2	Jan/95
Egypt	2.4	9.2	0.5	Jan/95	Household products	1.2	4.6	0.3	Jan/95
Hungary	2.2	10.0	1.3	Jan/95	Industrial conglomerates	1.1	5.1	1.1	Jan/95
India	1.5	8.2	0.7	Jan/93	Information technology services	0.3	7.8	1.3	Jan/95
Indonesia	2.0	15.0	0.9	Jan/88	Insurance	0.9	5.0	1.0	Jan/95
Israel	1.0	7.2	1.0	Jan/93	Internet catalogue & retail	1.1	9.3	1.2	Jan/95
Jordan	0.8	5.1	0.1	Jan/88	Internet software services	2.0	17.2	2.3	Jan/95
Korea	1.3	11.1	1.2	Jan/88	Leisure equipment & products	0.4	4.3	0.6	Jan/95
Malaysia	1.2	8.8	0.9	Jan/88	Machinery	0.8	5.1	1.0	Jan/95
Mexico	2.3	9.2	1.1	Jan/88	Marine	1.0	6.3	1.0	Jan/95
Morocco	1.5	5.3	0.2	Jan/95	Media	0.6	5.5	1.2	Jan/95
Pakistan	1.4	11.0	0.4	Jan/93	Metals & mining	1.3	6.6	1.1	Jan/95
Peru	2.1	8.9	0.8	Jan/93	Multi utilities	0.7	6.1	1.0	Jan/95
Philippines	1.1	9.4	0.9	Jan/88	Multiline retailers	1.0	5.2	0.9	Jan/95
Poland	2.6	14.7	1.6	Jan/93	Office Electronics	0.7	6.4	1.0	Jan/95
Russia	3.4	17.1	2.0	Jan/95	Oil, gas & consumable fuels	1.4	4.9	0.7	Jan/95
South Africa	1.5	7.7	1.1	Jan/93	Paper & forestry products	0.5	5.8	1.0	Jan/95
Sri Lanka	1.0	10.0	0.3	Jan/93	Personal products	1.3	5.2	0.8	Jan/95
Taiwan	1.2	10.9	0.9	Jan/88	Pharmaceuticals	1.0	3.9	0.5	Jan/95
Thailand	1.3	11.4	1.3	Jan/88	Road & rail	0.6	3.8	0.5	Jan/95
Turkey	2.6	17.3	1.3	Jan/88	Software	1.7	8.8	1.6	Jan/95
Venezuela	1.7	14.5	0.9	Jan/93	Specialty retail	0.8	5.9	1.1	Jan/95
					Textiles, apparel & luxury goods	0.9	5.3	1.1	Jan/95
					Tobacco	1.7	6.6	0.4	Jan/95
					Trading companies & distributors	0.7	6.7	0.9	Jan/95
					Transportation infrastructure	1.1	4.8	0.5	Jan/95
					Water utilities	1.6	5.0	0.3	Jan/95
					Wireless telecommunication services	1.5	6.7	1.1	Jan/95

Exhibit A2: GLS and Related Magnitudes

This exhibit shows, for the series of monthly returns summarized in Exhibit A1, the probability of a loss (p_L), the average loss (A_L), the average gain (A_G), and the gain-loss spread (GLS) for all the countries and industries in the sample, all calculated between the beginning and the end of each variable's sample period and for a benchmark of 0%. Returns are in dollars and account for capital gains and dividends. All numbers in %.

Country	p_L	A_L	A_G	GLS
Developed				
Australia	43.4	−4.6	5.4	5.1
Austria	42.5	−3.6	4.7	4.2
Belgium	37.7	−3.8	4.3	4.1
Canada	39.7	−4.0	4.4	4.2
Denmark	39.3	−3.7	4.5	4.2
Finland	44.2	−6.1	7.6	7.0
France	41.4	−4.5	5.2	4.9
Germany	41.0	−4.3	4.9	4.7
Hong Kong	40.6	−6.6	7.7	7.2
Ireland	41.3	−4.2	4.6	4.4
Italy	45.2	−5.1	5.8	5.5
Japan	44.1	−4.4	5.4	4.9
Netherlands	36.2	−3.8	4.1	4.0
New Zealand	44.6	−4.8	5.3	5.1
Norway	43.0	−5.3	6.4	5.9
Portugal	43.8	−4.6	5.0	4.8
Singapore	40.1	−5.5	5.9	5.8
Spain	42.8	−4.4	5.2	4.9
Sweden	41.9	−4.7	5.8	5.4
Switzerland	40.8	−3.5	4.4	4.0
UK	40.1	−4.3	4.8	4.6
USA	39.3	−3.1	3.6	3.4
Emerging				
Argentina	44.2	−9.0	12.0	10.7
Brazil	39.6	−10.0	11.7	11.0
Chile	40.8	−4.6	6.3	5.6
China	47.2	−7.8	8.2	8.0
Colombia	41.7	−6.8	7.9	7.4
Czech Rep.	37.2	−5.9	6.5	6.3
Egypt	41.0	−5.4	7.9	6.9
Hungary	40.4	−6.7	8.3	7.7
India	43.3	−6.1	7.4	6.9
Indonesia	43.3	−9.0	10.5	9.9
Israel	37.2	−6.1	5.2	5.5
Jordan	49.2	−3.1	4.5	3.8
Korea	50.0	−6.6	9.3	8.0
Malaysia	42.9	−5.9	6.4	6.2
Mexico	37.9	−6.8	7.9	7.5
Morocco	39.7	−3.4	4.7	4.2
Pakistan	50.6	−6.6	9.7	8.1
Peru	39.4	−6.1	7.4	6.9
Philippines	45.4	−6.6	7.5	7.1
Poland	40.6	−8.8	10.3	9.7
Russia	41.0	−11.4	13.6	12.7
South Africa	38.3	−6.0	6.1	6.1
Sri Lanka	46.1	−6.7	7.5	7.1
Taiwan	47.9	−7.3	9.0	8.2
Thailand	44.2	−7.9	8.6	8.3
Turkey	47.1	−11.2	14.8	13.1
Venezuela	51.1	−8.2	12.1	10.1
Average	**42.5**	**−5.9**	**7.1**	**6.6**

Industry	p_L	A_L	A_G	GLS
Aerospace & defense	36.5	−4.1	4.4	4.3
Air freight & logistics	41.7	−3.3	4.0	3.7
Airlines	43.6	−4.4	4.5	4.5
Auto components	42.3	−3.9	4.0	3.9
Automobiles	36.5	−4.5	4.0	4.2
Beverages	36.5	−3.2	3.3	3.2
Biotechnology	48.7	−5.3	6.9	6.1
Building products	43.6	−3.7	3.8	3.8
Chemicals	41.0	−3.2	3.9	3.6
Commercial banks	39.7	−3.6	3.9	3.8
Commercial services & supplies	38.5	−3.8	3.1	3.4
Communications equipment	42.9	−7.3	7.6	7.4
Computers & peripherals	44.2	−5.8	6.9	6.4
Construction & engineering	44.2	−4.0	4.7	4.3
Construction materials	37.8	−4.1	3.9	4.0
Containers & packaging	44.9	−4.1	3.9	4.0
Distributors	42.9	−6.8	5.2	5.9
Diversified financial services	37.2	−4.3	4.2	4.2
Diversified telecommunication services	39.1	−4.4	4.0	4.2
Electric utilities	35.9	−2.4	2.9	2.7
Electronic equipment & instruments	47.4	−5.4	5.8	5.6
Electronic equipment manufacturers	39.1	−4.3	4.4	4.4
Energy equipment & services	44.2	−5.7	7.4	6.7
Food products	34.0	−2.7	2.7	2.7
Food/staples retailing	36.5	−2.7	2.7	2.7
Gas utilities	37.8	−2.7	3.3	3.1
Health care equipment & support	35.3	−3.2	3.3	3.3
Health care providers & services	38.5	−4.4	4.2	4.3
Hotels, restaurants & leisure	39.7	−3.4	3.7	3.6
Household durables	44.2	−4.8	4.6	4.7
Household products	35.3	−3.3	3.6	3.5
Industrial conglomerates	39.1	−3.7	4.1	4.0
Information technology services	42.3	−6.2	5.0	5.5
Insurance	40.4	−3.5	3.9	3.7
Internet catalogue & retail	49.4	−5.8	7.8	6.8
Internet software services	42.3	−11.6	12.2	11.8
Leisure equipment & products	46.2	−3.3	3.6	3.4
Machinery	41.7	−4.0	4.3	4.2
Marine	42.9	−4.4	5.1	4.8
Media	37.2	−4.5	3.6	4.0
Metals & mining	41.0	−4.9	5.5	5.3
Multi utilities	34.6	−5.5	3.9	4.5
Multiline retailers	38.5	−4.2	4.2	4.2
Office Electronics	40.4	−5.4	4.9	5.1
Oil, gas & consumable fuels	34.6	−3.6	4.1	3.9
Paper & forestry products	47.4	−4.2	4.6	4.4
Personal products	32.1	−4.2	3.9	4.0
Pharmaceuticals	37.8	−3.0	3.4	3.3
Road & rail	42.3	−3.0	3.2	3.1
Software	39.7	−6.2	6.9	6.6
Specialty retail	38.5	−5.0	4.4	4.6
Textiles, apparel & luxury goods	42.3	−3.8	4.3	4.1
Tobacco	31.4	−5.8	5.1	5.3
Trading companies & distributors	45.5	−5.1	5.6	5.4
Transportation infrastructure	35.3	−4.1	3.9	4.0
Water utilities	37.2	−3.3	4.5	4.1
Wireless telecommunication services	35.9	−5.1	5.3	5.2
Average	**40.1**	**−4.4**	**4.6**	**4.5**

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